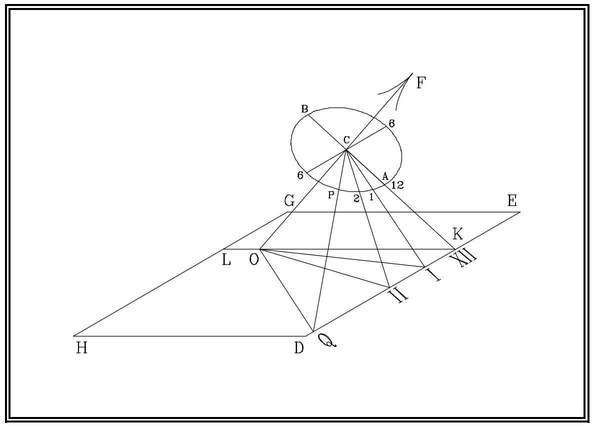
Geometrical Theory of Dialling.

18. It has been shown (art. 3 and 4) that if at any place a straight rod or wire be parallel to the earth's axis, in which position it may be considered as coinciding with the axis of the heavens, the angular motion of the sun about that rod will appear to be perfectly uniform throughout the day, and therefore the shadowy space within which the sun's light is wholly or in part intercepted by the rod will also turn uniformly about it. Hence, to construct a sun-dial, it is only necessary to place the rod so that its shadow may fall on a surface of any kind, and to trace on that surface, at any equal intervals of time (hours for instance), the line shown on the surface by the shadow at the instants which separate these intervals. These lines, numbered according to the hours, will serve to show time by the shadow at all seasons. This is the most simple way of constructing a dial, but it supposes that we have the means of dividing time into equal intervals, a thing indeed easy since the invention of clocks and watches.

To reduce the subject to a geometrical theory, let us suppose that, in the adjoining figure, *OCF* is the rod (considered as a material straight line) which projects the shadow, and that it is perpendicular to the plane of a circle *APB* at *C* its centre, and let *CA* be the position of the shadow on the circle at noon. Then, if the circumference be divided into twenty-four equal parts, beginning from *A*, as at *1*, *2*, *&c.*, and lines be drawn from the centre through the points of division, it is manifest that the shadow will fall on the lines at the hours marked on them, and the circle will serve as a dial. And since its plane coincides with that of the equator in the heavens, it is an EQUINOCTIAL DIAL.



Equinoctial Dial.

19. To construct this dial (Plate CCII. fig. 6), on *C*, a point in the middle of its face, as a centre, describe a circle *ABDE*; divide the circumference into twenty-four equal parts, and from the points of division draw straight lines to the centre; these will be the hour lines, mark the hours on them as in the figure; fix a thin and straight wire in the centre, perpendicular to the face of the dial, for its style, and place it with the style directed to the pole, and the twelve o'clock hour line in the plane of the meridian; and when the dial is illuminated by the sun, the hours will be indicated by the shadow of the style.

20. In our climate the superior side of an equinoctial dial is illuminated when the sun is on the north side of the equator, and the inferior or opposite when he is on the south side. On the equinoctial days neither side will be illuminated, because the sun is in the plane of the dial. However, if it have a ledge

Quadrant published Anno 1638... The reference here is to Samuel Foster's *The Art of Dialling*, London, 1638. In Foster's work, the scale of Latitudes is laid down simply by reference to a table of values; there is no derivation of the numbers. Foster's derivation of the table was found in his notes after his death and incorporated into a Latin text, *Demonstratio Quadrantis Horometrici* (*Demonstration of an Horometrical Quadrant*), published in 1659 as part of Foster's *Miscellanies* by his friend John Twysden.

rising a little above the opposite sides, and the hour lines be continued on the ledge, the shadow will fall on its inside, and indicate the hour, although the direction of the sun's rays be almost parallel to its face.

21. To set up an equinoctial dial, direct the straight edge of a vertical plane towards the polar star, which is about one degree thirty-six minutes from the pole of the world; it will then nearly coincide with the axis of the sphere; but for greater accuracy the edge may be directed to the star when highest or lowest, and a line drawn on the plane, making with its edge the above angle. This will be in the true direction of the axis of the dial, the plane of which must be placed perpendicular to the line so determined, and the six o'clock hours in a horizontal line: the dial will then be properly placed.

22. An equinoctial dial may be set up about the time of either solstice without knowing either the latitude of the place or the direction of the meridian, from this property, that when truly placed, the extremity of the shadow of the axis will then describe a circle on the plane of the dial, the centre being the common intersection of the hour lines. If, therefore, the dial be placed nearly in a true position, with the six o'clock hour line exactly horizontal, by observing the line which is the extremity of the path of the shadow, it will be seen in which way the deviation from the true position lies, and by repeated adjustment it may be truly placed. About the equinoxes the daily path will deviate somewhat from a circle, by reason of the quick change in the sun's declination.

Horizontal Dial.

23. Let *GEDH* (see the adjoining figure) be a horizontal plane on which a dial is to be delineated, and let *LCK* be a meridian line on this plane; let *OCF* be a material line or rod in the plane of the meridian, which meets the horizontal plane in *O*, and makes with *OK* an angle equal to the latitude of the dial: this rod will be directed to the pole, and will be the axis of the dial. Let *BAP* be an equinoctial dial, having *OCF* for its axis, and *C* for its centre; and let *CA*, the meridian line on this dial, meet the meridian line on the horizontal plane in *K*. As has been explained, the plane of the shadow will turn uniformly about the axis *OCF*, meeting the equinoctial plane in some line *CPQ*, and the horizontal plane in a corresponding line *OQ*. Let *C1*, *C2*, *&c.* be the hour lines after noon on the equatorial dial, and *O* I. *O* II. *&c.* the correponding hour lines on the horizontal dial, the former will make with the meridian line *OAK* angles proportional to the time from noon, and will be known when the hour is given, 15 degrees being reckoned an hour. The plane of the equinoctial dial being supposed extended to meet the horizontal plane in the line *QK*, which will be at right angles to the meridian lines *CK*, *OK*, the problem to be now resolved is, to find the hour angle *KOQ* at the centre of the horizontal dial corresponding to any given angle *KCQ* at the centre of the equinoctial dial, which measures the time from noon.

The triangles *CKQ* on the equinoctial plane, and *OKQ* on the horizontal plane, have a common side *KQ*, and each a right angle at *K*; therefore, by trigonometry,

$$CK/KQ = 1/\tan KCQ,$$

$$\text{and } KQ/OK = \tan KOQ,$$

$$\textit{therefore, } CK/OK = \tan KOQ/\tan KCQ.$$

But in the triangle *COK*, right angled at *C*,[29]

[29] The original text confuses angle *COK* with angle *OCK*.

$$CK/OK = \sin COK,$$
therefore $\sin COK = \tan KOQ/\tan KCQ$.

Hence we have this general theorem or rule for computing the hour angles at the centre of a horizontal dial: *The sine of the latitude equals the ratio of the tangent of the hour angle at the centre of a horizontal dial to the tangent of the hour from noon (reckoning 15º to an hour)*; or, putting *x* for the hour from noon in degrees, *y* for the hour angle at the centre of the dial, *L* for the latitude of the place,

$$\tan y = \tan x \sin L \qquad (1)$$

We have supposed *GHDE* to be a horizontal plane, but the formula evidently applies to any plane whatever perpendicular to the meridian: all that is required for its application is the angle which the axis *OF* makes with the meridian line *OK* on the plane.

As an example, let it be required to find the hour angle at the centre of a horizontal dial for XI. or I. o'clock for the latitude of London 51½º. In this case the hour angle from noon at the pole is 15º.

$$\tan y = \tan 15^\circ \sin 51^\circ 30'$$
$$= .26795 \times .78261$$
$$= .20970$$
$$\text{so, } y = 11^\circ 51'$$

Here we have found that the hour lines of XI. and I. must each make an angle of 11º 51' with the meridian line at the centre of the dial.[30]

Geometrical Construction of a Horizontal Dial.

25. The formula of art. 23, namely, that the tangent of the angle which any hour line makes with the meridian line is a fourth proportional[31] to radius, the sine of the latitude, and the tangent of the hour angle at the pole (that is, the hour from noon in degrees), reduces the construction of a dial to this geometrical problem.

Having given any angle, to find another whose tangent shall have to that of the former a given ratio. This problem may be resolved graphically in various ways, and in as many ways may the hour lines on a dial be determined.

First Construction.

26. Draw two parallel straight lines *CM*, *C'M'* (Plate CC. fig. 5) at a distance equal to the thickness of the style for the double meridian line, and cross them at right angles by the six o'clock hour line VI. *C'C* VI. Make a right angled triangle *HCK* (fig. 6), having *K* a right angle, and *C* equal to the angle which the axis of the dial is to make with its plane, that is, to the latitude of the place, which for London is 51½º. About *C* and *C'* (in fig. 5) as centres with a radius equal to *CH*, the hypotenuse of the triangle *CKH* (fig. 6), describe the quadrants *M6*, *M'6*; and about the same centres, with a radius equal to *HK*, the side of the triangle opposite to *C*, describe concentric quadrants, as in the figure; divide these each into six

[30] This section is followed in the original by section 24, which provided a table of the angles which the hour lines make with the meridian, computed for every half degree of latitude from 50º to 59º 30'. This section has been omitted here.

[31] A quantity *d* is a fourth proportional to quantities *a*, *b* and *c* if $a/b = c/d$; or equivalently, $d = bc/a$. The 'radius' in this example is 1.