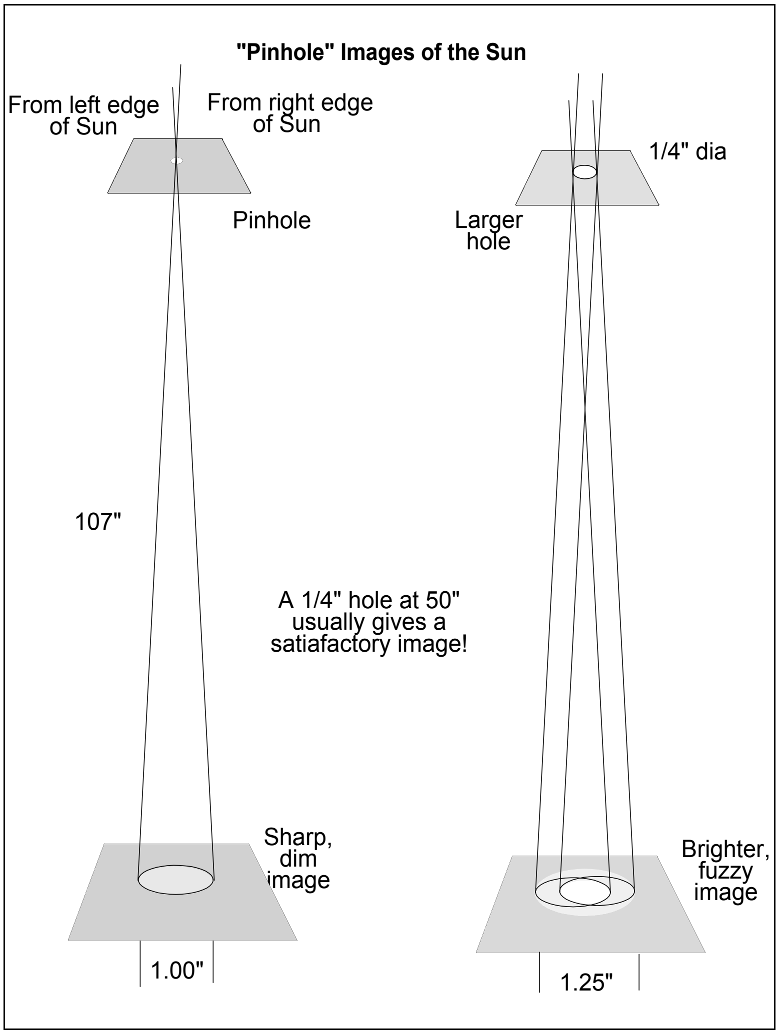
Pinholes and Shadow Sharpeners

William Walton (Plymouth MA)

Introduction

After repeated e-mail messages on the Sundial Mailing List, Mac Oglesby wrote to several of us asking for an article that would clarify what was meant by a “shadow sharpener,” (how can you sharpen a non-material entity like a shadow?), and describe some experiments that would show what was possible in reading the position of a shadow more precisely. This article is my response to that request.

I want to thank all of the Sundial Mailing List contributors who shared their insights on pinholes and shadow sharpeners. Among those were Roger Bailey, Art Carlson, John Carmichael, Gianni Ferrari, Bill Gottesman, and Pete Swanstrom.



I divide the topic into two main parts. First, “Pinholes” which can be used to locate more precisely the geometric center of certain shadows, and “Shadow Sharpeners” which, in spite of the conceptual difficulty with the term, really do make the center of a diffuse shadow more sharply defined.

Pinholes

“Pinholes,” in our use, may vary from “darning needle holes” to “knitting needle holes,” but are used here to make it possible to find very precisely the geometric center of a penumbral shadow.

A Pinhole Image of the Sun

On the left in the diagram we see a pinhole image of the Sun. A beam from the left hand edge of the Sun travels straight through the pinhole and make the right edge of the image. A beam from the right edge of the Sun makes the left edge. Between the right and left edges, beams from the remainder of the Sun fill in the image.

The image is sharp but dim because so little light can come through the small pinhole. Since the angular width of the Sun is 32 minutes, the resulting diameter of the Sun’s image is 1/107th the distance from the hole to the screen

On the right we have a larger hole, say ¼ inch in diameter. Now the bright area on the bottom screen is made up of overlapping images of the Sun. In theory, it will consist of a central bright area ¾ inch in diameter surrounded by a fuzzy edge ¼ inch wide. In practice, the fuzzy edge is much narrower due to the logarithmic sensitivity of the eye to light. This results in sharper perceived image than depicted in the

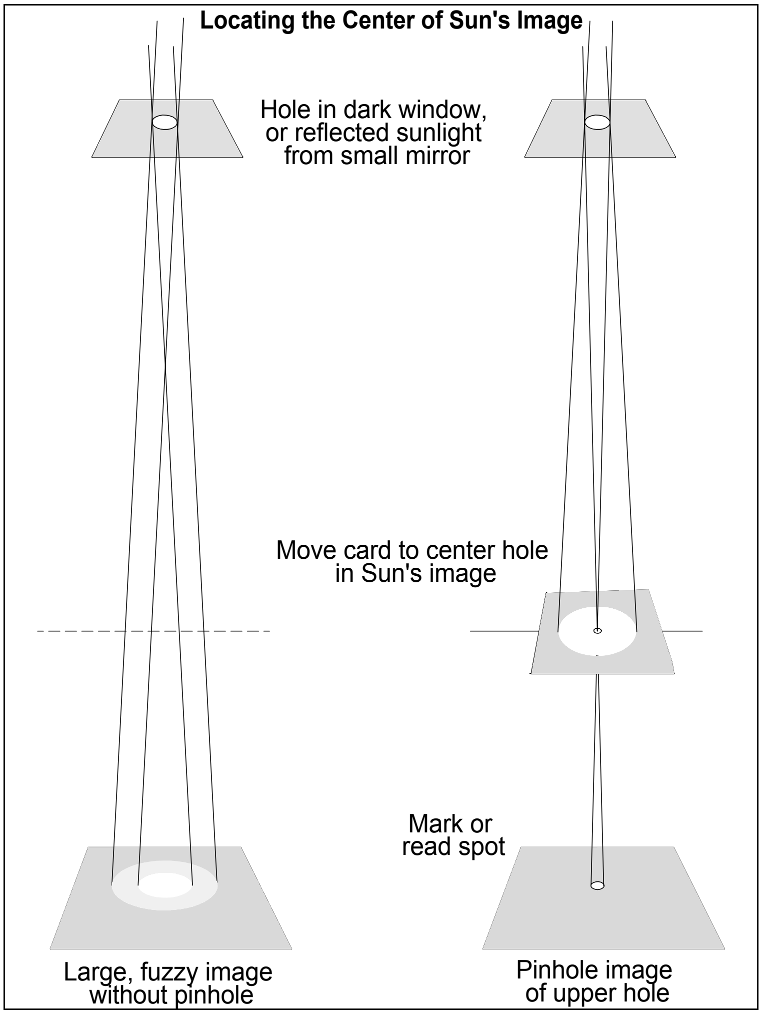


diagram. Experience shows that a ½ inch hole at 107 inches, or a ¼ inch one at 50 inches gives a satisfactory sharp, bright image of the Sun.

Locating The Center of the Sun's Image

Pinhole images of the Sun are found in noon marks, meridians in cathedrals, and reflected ceiling dials. [slide] Often it is difficult to find the center of the large, fuzzy spot of light on a sloping surface. The diagram on the right shows a remedy for this situation. Take a small card and punch a secondary pinhole in it (1/16th to 1/8th inch diameter usually works well). Hold this card, with the hole centered in the Sun's image, a few inches above the surface. The narrow shaft of light that passes through forms a pinhole image of the upper hole in the center of the original beam. Subjective judgment is still required to center the hole in the Sun's image, but since this image may be made circular by holding the card perpendicular to the beam the eye can do a surprisingly good job of finding its center.

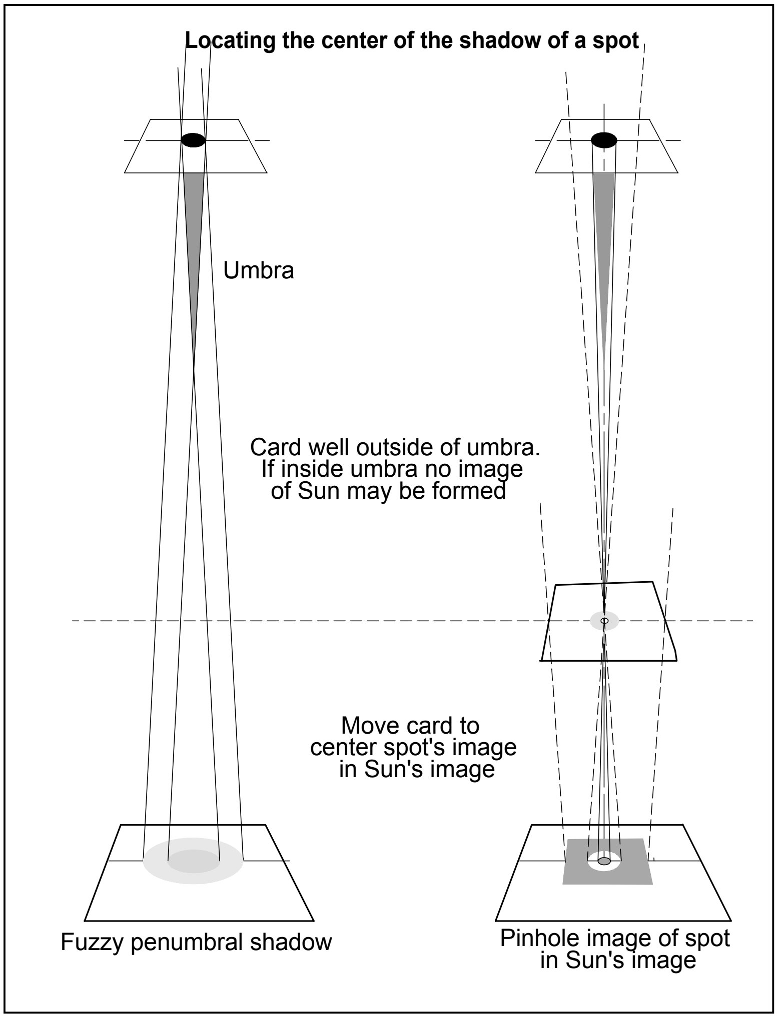
Locating the Center of the Shadow of an Opaque Spot

The diagram shows on the left an opaque spot with its umbra and penumbra in the light from the Sun. A card held just inside the tip of the umbra will show a dark spot surrounded by a gradually fading penumbra. This shadow may be sharpened by use of a special mask known as a "shadow sharpener" and discussed later in this paper.

To achieve greater angular resolution it is often desired to locate the scale on which the shadow falls beyond the tip of the umbra. On the right, the diagram shows a card with pinhole placed in the penumbra of the shadow. Here the card forms a pinhole image of the sun with the image of the spot superimposed upon it. The card with pinhole may be moved about until the image of the spot is centered in the image of the sun. At this point the image of the spot marks the center of the shadow of the opaque spot. Here the advantages of this method are real. The greater angular resolution of a distant scale may be taken advantage of, the image of the spot would not be visible at all without the card with pinhole, and the small image of the spot may be centered very precisely in the small image of the Sun.

Locating the Center of the Shadow of an Opaque Band

Here the opaque band may be the wire, rod, or pipe used as a gnomon for an equatorial dial. When the scale is near the tip of the umbra or well within the umbra, a broad to narrow dark shadow may be formed. The edge of the broad shadow may be read, but see the next section for precautions in reading



the edge of a shadow. If the narrow shadow at the tip of the umbra is used, a "shadow sharpener," discussed later, may be used.

Again, to achieve greater angular resolution it is often desired to locate the scale on which the shadow falls beyond the tip of the umbra. On the left we see the dim, fuzzy penumbra of the band. On the right, the diagram shows a card with pinhole placed in the penumbra of the shadow. Here the card forms a pinhole image of the sun with the image of the band, as a sharp line, super-imposed upon it. The card with pinhole may be moved about until the line is bisects the image of the sun. At this point the line marks the center of the shadow of the opaque band. Again, the advantages of this method are real. The greater angular resolution of a distant scale may be taken advantage of, the image of the band would not be visible at all without the card with pinhole, and the small image of the band may be centered very precisely in the small image of the Sun.

Locating the Geometric Center of the Shadow of an Edge

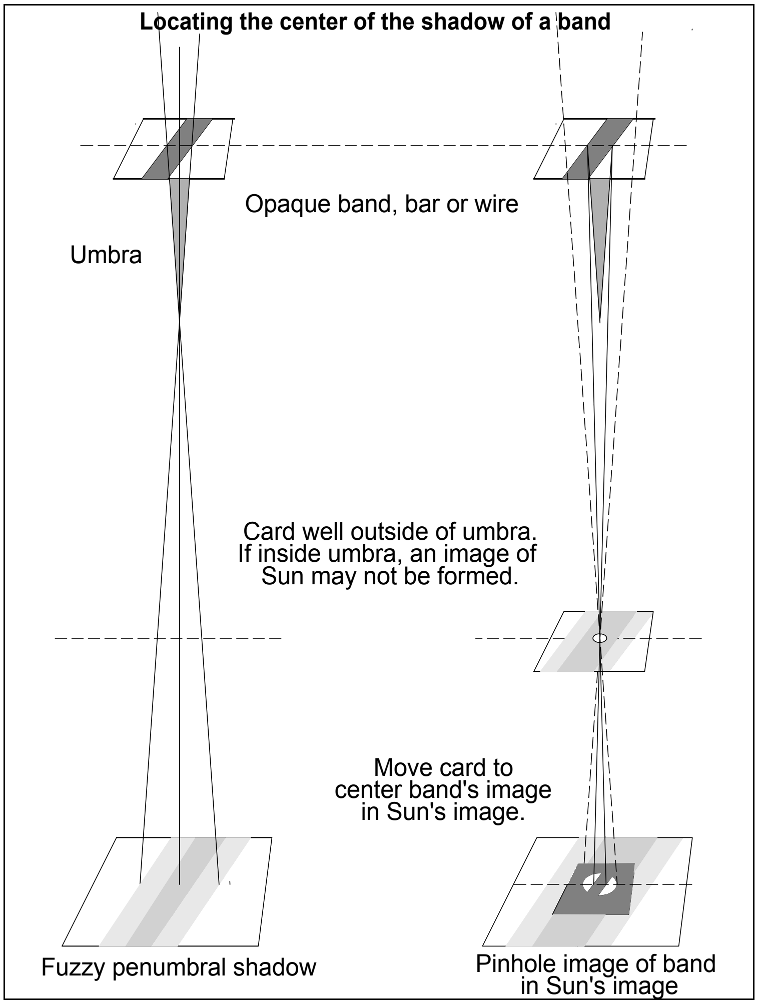
The diagram on the left shows the penumbra formed by the shadow of an edge. The umbra would be the dark area to the right of the fuzzy penumbra. The center of the geometric shadow of the edge is in this penumbra.

The diagram on the left shows how to precisely locate the center of this penumbra using a card with a pinhole in it. The pinhole forms an image of the Sun with an image of the edge superimposed upon it. The pinhole is moved in and out of the penumbra until the image of the sun appears as a half-circle. At this point the diameter of the half-circle marks the geometric center of the shadow of the edge.

Logic would tell us that at the geometric edge of the shadow the illumination would be midway between the illumination of the un-shadowed card and the illumination of the deep shadow. Experience shows that this is not what the eye perceives. The eye sensitivity to light is non-linear and varies approximately with the logarithm of the illumination. This throws the perceived mid-shadow toward the umbra, making the fuzzy edge of the shadow appear much narrower than shown in the diagram. The perceived edge of the shadow might appear to be moved roughly ¼ the width of the penumbra away from the geometric center of the shadow. Since the angular width of the penumbra is the same as the angular width of the Sun, which is one-half a degree of arc or two minutes of time, the error in reading the shadow of an edge might be approximately one-quarter of two minutes, or 30 seconds of time.

Shadow Sharpeners

Shadow Sharpeners really exist, and are used to make more narrow the somewhat diffuse shadow of a small object.



Shadow Sharpener for an Opaque Spot

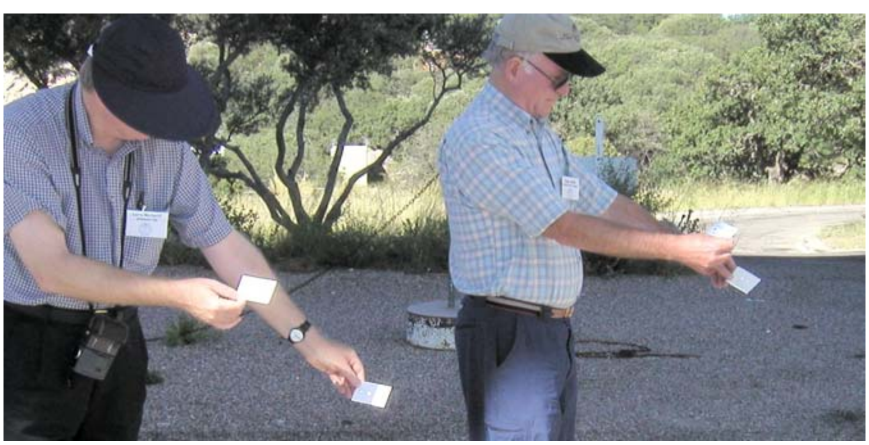
After a series of experiments to determine the best configuration for an alidade to be used in a helio-chronometer, John Carmichael (4/11/00) found that a 1/8 inch bead surrounded by a ¼ inch hole in opaque material provided a sharper shadow on a scale 18 inches away than the 1/8 inch bead did alone. The following diagram by Bill Gottesman explains how this so-called shadow sharpener works to provide a more narrowed central dark shadow.

Shadow Sharpeners for an Opaque Band and "Cross Hairs"

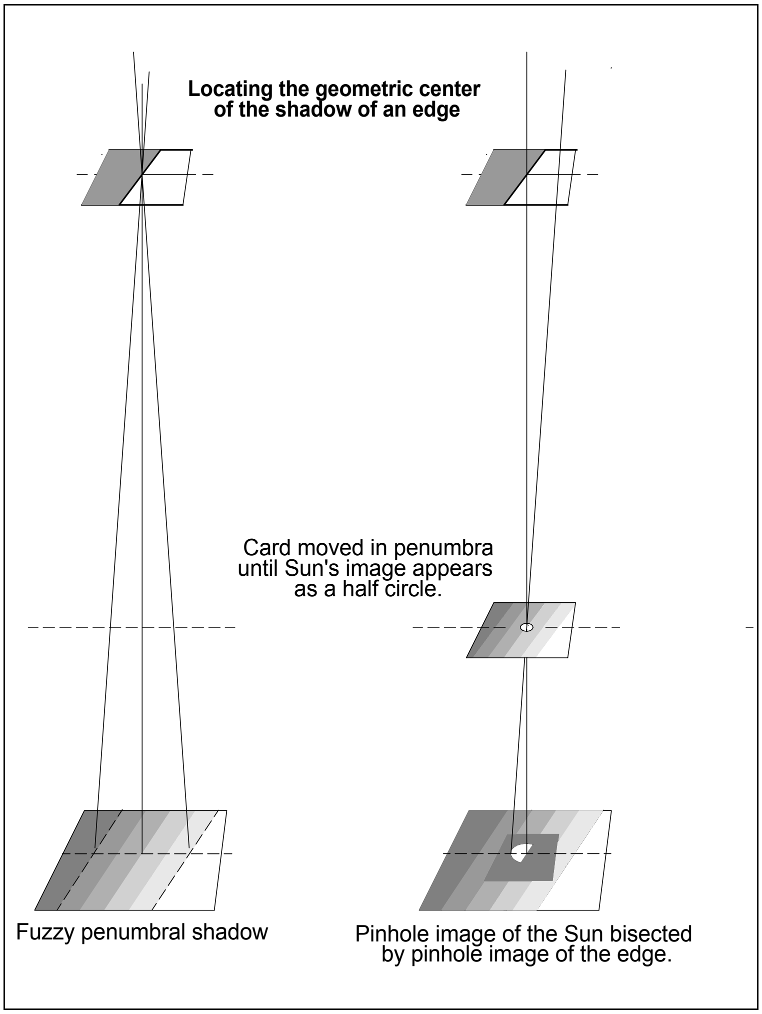
The idea for a shadow sharpener for a spot can be extended to a band or even to "cross hairs" as the following diagram and slides show.

Conclusion

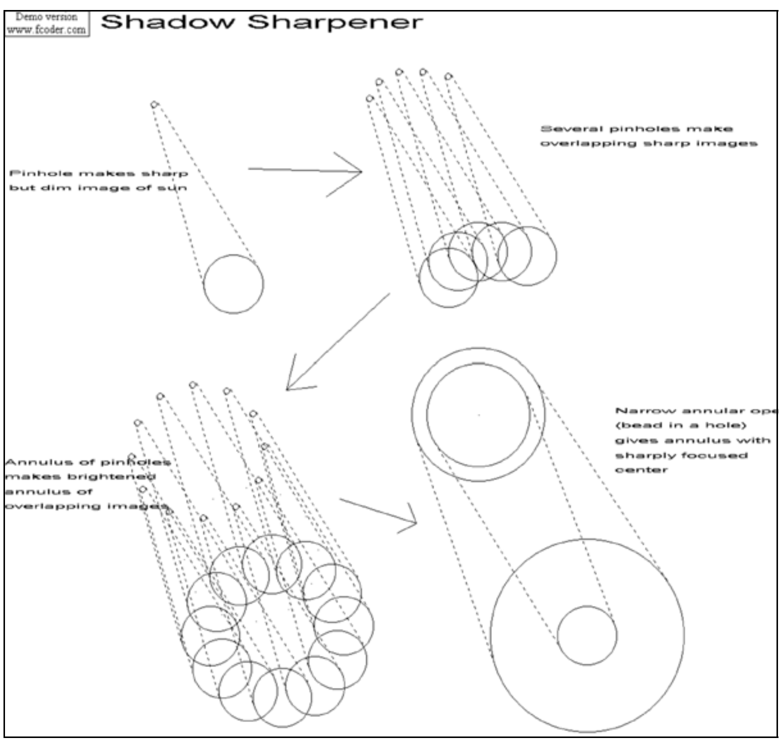
I have shown that a card with a "pinhole" in it may be used to find the geometric center of a penumbral shadow cast by an opaque spot, band or edge at a relatively great distance from the shadow-casting object. A complementary technique may be used to find the center of an image of the sun, but is more subject to errors of judgment. These methods are applied to a sundial by the use of a hand-held card, and so are note useful for the casual observer. It should be possible to develop an equatorial dial with a fixed pinhole (or narrow slit) at noon (or any other time or times) that would enable the set time to be read with a precision of a few seconds. Perhaps a series of diagonal slits could be used to read the dial at any arbitrary time.



Experimenting with shadow sharpeners at Kitt Peak during the NASS Tucson conference.



William Walton
15 Russell Avenue
Plymouth MA 02360
wuwalton@aol.com



The Shadow Sharpener

Gianni Ferrari (Modena, Italy)

A Shadow Sharpener is a tool that helps us see the separation line between the shadow and the penumbra produced by a distant object illuminated by the Sun. A tool, in other words, that allows us to mark with good approximation where the "geometric" or "theoretical" shadow of a distant sunlit object finishes - the place where the line between light and shadow would be if the Sun were a point source of light.

A shadow sharpener can be made in different ways, including devices with complex optical systems, but the simplest and most ancient sharpeners are based on the projection, made with a simple little hole, of the image of the disk of the Sun and of the object that casts the shadow [1]. Devices of this type were probably used by the Chinese astronomers many centuries ago, and certainly by the Indian astronomers at the beginning of 1700's to find the edge of the gnomon's shadow in the great equatorial sundial of Jaipur in India.

The device is very simple, consisting of a piece of opaque material containing a little circular hole having a diameter from about 1/2 mm to 2 mm. A simple and practical shadow sharpener may be made by using a large needle or a nail to punch a hole in a playing card or rectangular bit of metal cut from a "tin" can.

If we place a shadow sharpener in such a way that its plane is perpendicular to the rays of the Sun, the hole produces a picture of the Sun's disk on an image-receiving screen (Fig. 1).

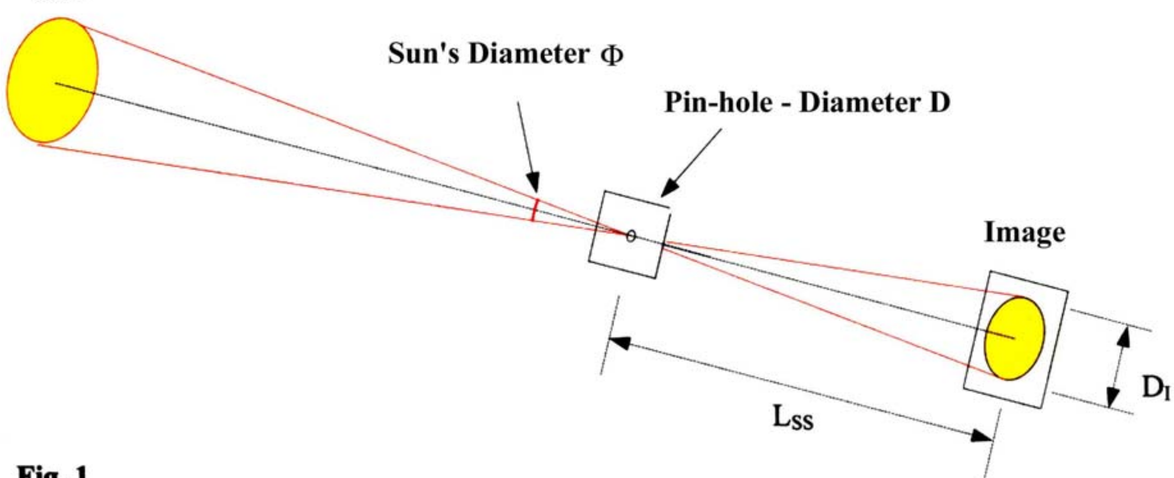


Fig. 1

Then if we call L_{SS} the distance between the hole and the screen image, D the diameter of the hole and Φ the angular diameter of the Sun in radians, the value D_I of the diameter of the image is given by $D_I = L_{SS} \cdot \Phi_{rad} \cong \frac{L_{SS}}{107}$ in which $\Phi_{rad} \cong \frac{32}{60} \cdot \frac{\pi}{180} \cong \frac{1}{107}$.

---

[1] A hole of this type is called pin-hole in Anglo-Saxons countries and "stenopeic hole" in Latin countries and in optics. The term stenopeic derives from the Greek word "steinôpos", formed by the two words "stenos" + "opê" that, translated literally, mean "narrow" and "opening, hole" (two words that have the same root are e.g. "stenosis" and "operculum) and therefore "stenopeic hole" means literally "hole with a small opening" or also "hole of small diameter".

A simple mnemonic rule: the diameter of the Sun's image increases 1cm for every meter of the distance L_{SS}. The distance L_{SS} is often called the length of the shadow sharpener.

If the diameter D of the hole were close to zero, the image produced would be a very dim perfect copy of the projected object. Since the hole has finite dimensions we may assume that every point of its surface produces a projection of this type. We will get the resultant image by adding these innumerable elementary images. For this reason the image is always "defocused" and surrounded by a zone of uncertainty, or fuzziness, as wide as the hole diameter D.

If our light source is the Sun, and if we want the image to be sharp enough, it is necessary that that D_I, which is approximately equal to $L_{SS}/107$, be much greater than D, and therefore, that the shadow sharpener length L_{SS} is much greater than 107 times D. For most applications it suffices that D is approximately in the range of 1/400 to 1/300 of L_{SS}. Thus, $L_{SS}/400 < D < L_{SS}/300$.

Let us now suppose there is a blocking element with a straight edge set between the Sun and the shadow sharpener at a definite distance from it. This element can be a thread or a cable, the upper edge of a wall, the edge of a roof, the edge of a tilted gnomon, the style of a very large sundial, etc. Because of the apparent diameter of the Sun, between the blocking element's shadow and the zone in full light there is a zone of penumbra whose width is $L_R \cdot \Phi_{rad}$, where L_R is the distance between the blocking element and the shaded plane. (Fig. 2)

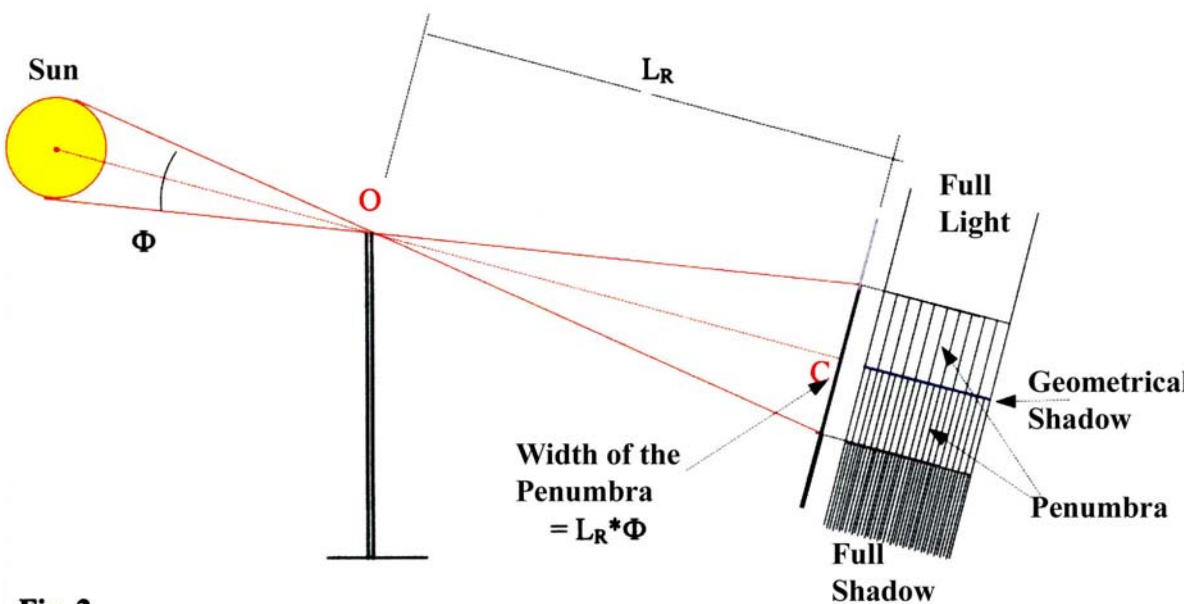


Fig. 2

If a shadow sharpener is placed to intercept the edge of the blocking element's shadow, and if the distance L_R is large in comparison to L_{SS}, then the hole produces both the image of the outline of the blocking element and that of the partially covered solar disk. (Fig. 3)

Holding the shadow sharpener in one's hand and moving it from the full light toward the full shaded zone, we see first the Sun's disk completely illuminated, then, as the hole enter the penumbra, the image of the Sun's disk that slowly becomes obscured. (Fig. 4)

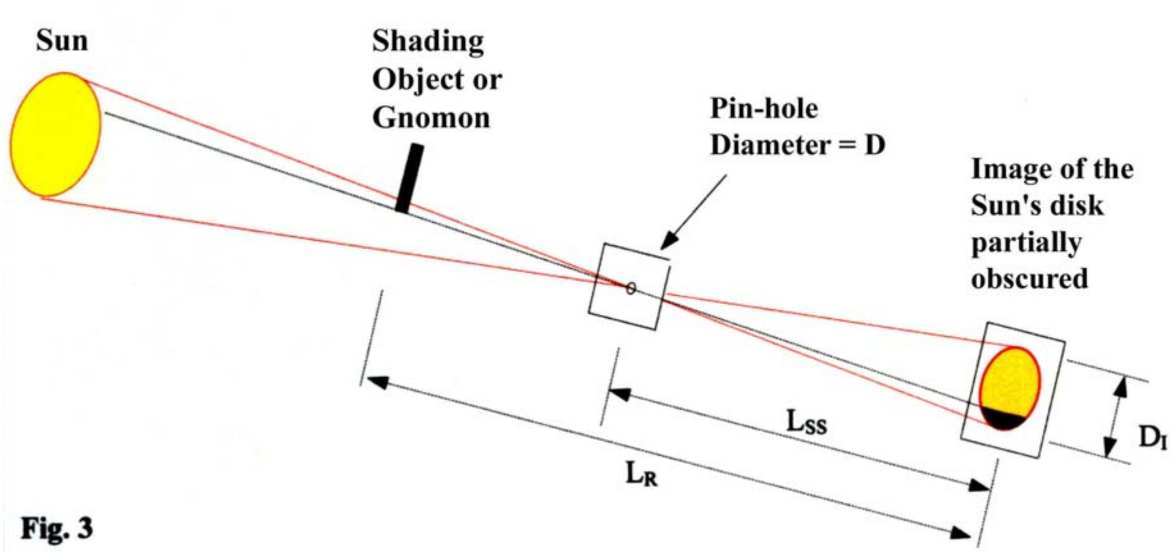


Fig. 3

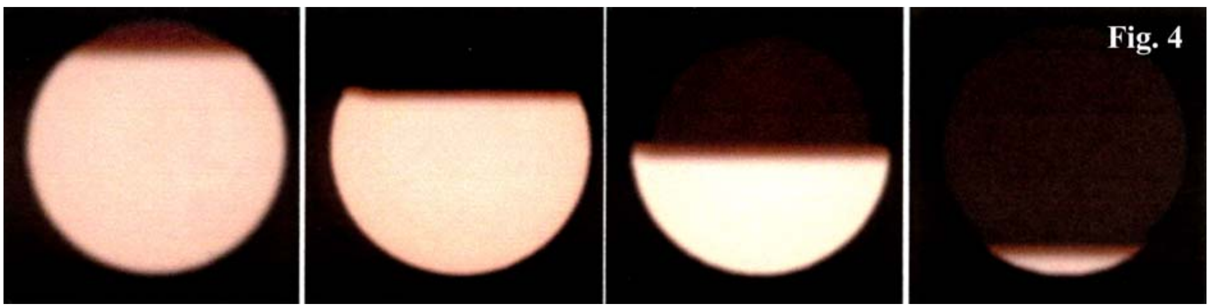


Fig. 4

When the Sun's disk appears exactly halved (3rd frame of fig. 4) the separation line between shadow and light coincides exactly with the line of the geometric shadow of the blocking element at the instant of the observation. This is because, by definition, the geometric shadow is what would have been produced by the Sun if its diameter were zero, and all of its light came from its center.

The search for the geometric shadow

The search for the geometric shadow of a linear element, particularly of the edge of a gnomon, is perhaps the main application of the shadow sharpener:

- either to verify the exactness of the hour lines of a sundial already drawn; i.e. to see if existing hour lines are properly located,

- or to find where the hour lines need to be located

In the first case we have to hold the shadow sharpener in such a way as to project the Sun's image, exactly half blocked by the gnomon, onto one of the lines (for example the hour line H) and to note the time. This is the exact instant at which our sundial says it is the hour H.

In the second case we move the shadow sharpener slowly and carefully, trying to keep the image of the Sun's disk continually halved while time passes, and then to mark on the plane the separation line between shadow and light exactly at the hour H.

The length L_{SS} of the shadow sharpener is limited by two different requirements:

- to be able to see comfortably and to tell with sufficient accuracy when the Sun's disk is exactly halved, it is necessary that image diameter D_I be 10 mm or more;

- to get an image of the element that casts shadow sharp enough (that is only slightly defocused), the length L_{SS} must be only a fraction of the distance L_R between the element and the screen (Fig. 3)

Several tests by some dialists have found that the length L_{SS} should be between about 70 cm and 150 cm. These values require a distance L_R of about 3m to 5m between the blocking element/gnomon and screen, and give an image of the Sun of 7 to 14 mm. Therefore a shadow sharpener cannot be used for making readings with sundials of usual dimensions, in which the distance between the gnomon and the plane is always a lot less than 2 or 3 m. L_{SS} would then have a length less then around 40 cm, giving a solar image of about 4 mm, not very useful in practice.

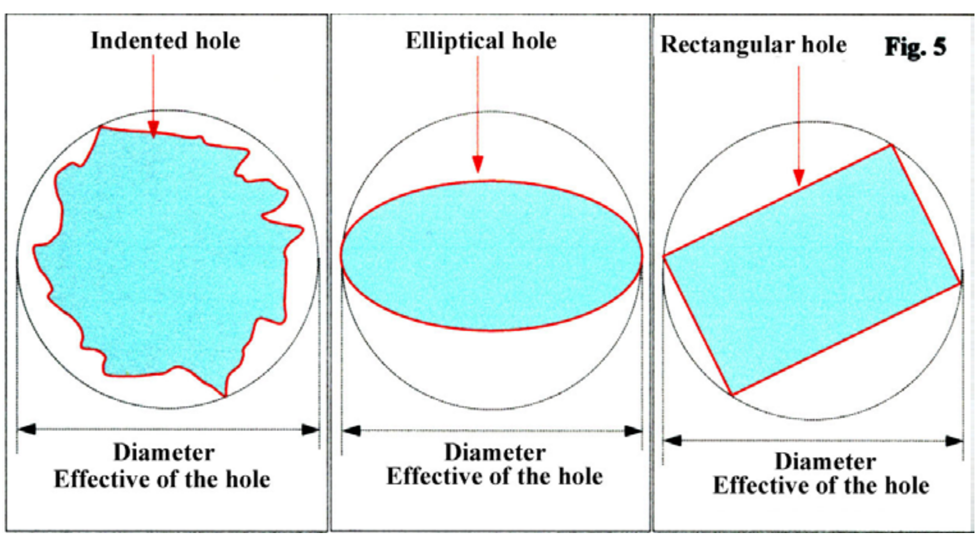
Errors - Accuracy of the measures.

Even if a shadow sharpener allows us to find with good approximation the line of the geometrical shadow of a gnomon at a given instant, it is also a source of errors and does not allow us to reach such an extreme accuracy as some think. The errors can be different and I list only the most important.

1. The image produced by a pin-hole is always surrounded by a strip of uncertainty (defocusing or fuzziness) that has the width of the maximum dimensions of the hole. This zone produces a reduction in the quality of the image with resulting difficulty in the search for the center line of the disk

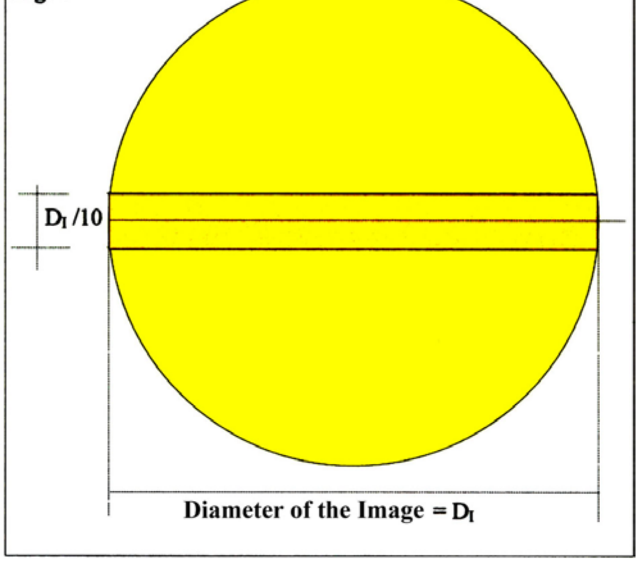
2. The dimension of the hole of a sharpener cannot be reduced beyond a fixed limit, either because the image loses sharpness due to the diffraction of the light, or because the image becomes too dim. The examination of this image becomes very difficult if it is only a little brighter than the surrounding zone. Many tests show that it is not useful to decrease the hole diameter under 0.7 - 1 mm

3. If the hole of the shadow sharpener has irregular edges, it produces a defocused zone as wide as the maximum diameter of the teeth. Likewise if the hole is not perfectly circular the defocused zone is, in some points, as wide as the maximum dimension of the hole (Fig. 5). It is for this reason, in addition to construction convenience, that the hole is always made circular. From the theoretical point of view its shape doesn't have any importance



4. To find with accuracy the instant at which the shadow of the element-gnomon halves the Sun's disk, it would be useful to have an image as large as possible and therefore it would be good to increase the length L_{SS} of the shadow sharpener. However as L_{SS} increases, it becomes more difficult to have a motionless image, and the image becomes dimmer (see item 2 above). Since the distance L_R between the element-gnomon and the plane has to be at least 5 - 8 times the distance L_{SS} (Fig. 3), L_{SS} cannot have very large values. If the ratio L_R / L_{SS} decreases, the defocusing of the image of the gnomon increases.

Fig. 6



5. In any case, the search for the instant at which the image of the solar disk is halved is not easy. Also supposing an uncertainty of 1/10 of the diameter (Fig. 6), in my opinion optimistic, the error at the instant would be $\pm\frac{1}{20}\cdot 2\,\text{min} = \pm 6\,\text{sec}$, where 2 minutes is the time needed by the Sun to cover a distance equal to its diameter[2]

Due to the reasons listed I do not believe that the total error, using a shadow sharpener with a pin-hole, at the instant in which the image of the Sun is halved, that is, at the instant in which the geometric shadow passes across a given point of the plane, can be less than ± 8 to 10 seconds.

<u>Some curiosities</u>

Besides the uses described, a pin-hole or a shadow sharpener allows us, for instance, to see on the ground the shadow of a wire or to see on a wall the shadows of tree leaves and branches, interspersed with spots of light. It is enough to put the hole at a distance from the wall, in the zone of shadow, and to move it slowly and with some patience. (Fig. 7)

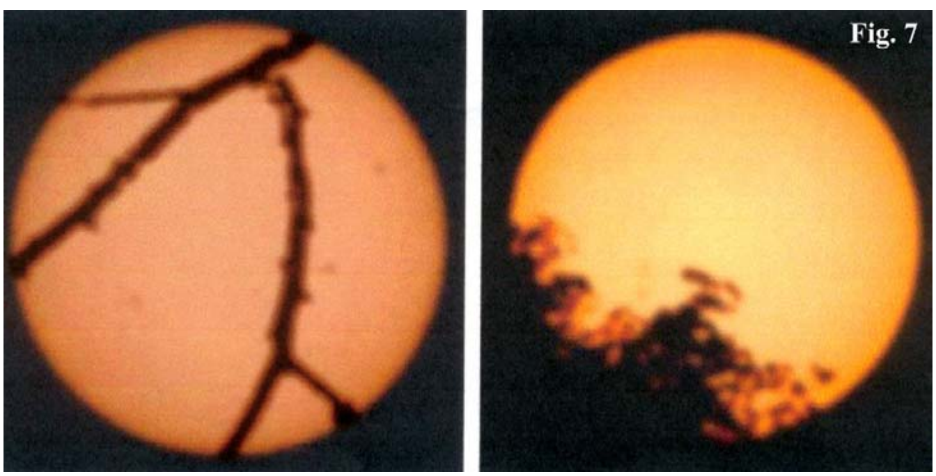


Fig. 7

[2] Since 1° of hour angle correspond exactly to 4 minutes of time, and the mean diameter of the Sun is around 31 ', it will take $\cong \frac{31}{60}\cdot 4\,\text{min} = 124\,\text{sec}$.

If you have ever stood beneath a tree during a partial eclipse of the Sun, you surely noticed the myriad of tiny crescent images of the partially blocked Sun, formed as sunlight filtered through the leaves. One tree, but a hundred or more shadow sharpeners.

A very simple shadow sharpener can be made by forming a hole with the fingers of the hands. For instance the fingertips of thumb and index of both hands can be joined and then moved nearer. In a different way, we may first close one hand and then lift the index finger, keeping it folded. Between it and the middle finger we obtain a small hole suitable to be used as a simple shadow sharpener.

Since a shadow sharpener allows us to "see" the profile of an obstacle against the background of the disk of the Sun, it cannot be used in pin-hole sundials, that is in the great sundials built in closed rooms or churches in which a little hole projects the image of the Sun. In these sundials a shadow sharpener would give only a small image of the hole without furnishing the image of the Sun's disk. If we could put our eye in the place of the hole of the sharpener we would see in fact only the hole of the sundial completely illuminated by the Sun, whose disk, if it were not partially covered, would appear much greater than the hole itself.

Different methods

An optical shadow sharpener can be made simply by projecting the image of the Sun with common binoculars. The image in this case is magnified and very clean and precise. With this "apparatus" we must use a stand or tripod that holds the binoculars steady and eliminates the inevitable trembling of our hands.

A different system to get the same results as a shadow sharpener, proposed by different members of the Sundial List in 1999, would be to put our eye "behind" or "on" the hour line or "behind" the plane of the sundial, and from this point to look toward the Sun. If we could do this we would see, if our eye were "behind" an illuminated zone, the full disk of the Sun moving slowly toward the outline of the gnomon, and finally, completely darkened, or covered, by it - exactly as happens with a shadow sharpener, except with a sharpener the image will seem left to right reversed. For putting our eye "on" the hour line it is enough to lean a small mirror on it and to look for the reflected image of the Sun, as William Maddux proposed for the first time on 16 May 1999. At the beginning this may take some patience.

Obviously it is necessary to diminish the brightness of the Sun using a welding glass or a pair of "eclipse glasses" or a solar filter in mylar or two or more layers of blackened photographic film (not exposed and developed). WARNING!! You must NEVER look at the Sun, or its reflection, without adequate eye protection.

Bibliography

Sundial Mailing List - Messages exchanged in 1999, 2000, 2001, and 2002. See the Sundial Mailing List archives at: http://groups.yahoo.com/group/sundial/messages or http://www.astroarchive.com

My heartfelt thanks to Mac Oglesby, who has helped me with the translation into English and has corrected, with a lot of patience, my numerous mistakes

Gianni Ferrari gf_merid@virgilio.it
Via Valdrighi 135, Modena 41100 Italy

[Editor's Note: The next issue of *The Compendium* will include another article by Gianni Ferrari on a closely related subject: *The Shadow and Penumbra of a Rectilinear Element*.]