Serle's Dialing Scales

Fred Sawyer

[Note: This article is excerpted from the talk the author gave at the annual conference in Washington DC. See the Special Offer page for an opportunity to obtain your own copy of the book and metal dialing ruler discussed here and prepared especially for NASS.]

Let me tax your imaginations for a few minutes. Suppose you find yourself in the English countryside, making your way towards London. And because this happens to be the mid-17th century English countryside, you notice sundials everywhere you look. You see them on farmhouses, at crossroads, on church walls - and even your fellow travelers carry portable dials with them.

All of this exposure inspires you to want to design and build your own sundial. Unfortunately, you don't have the benefit of any training in the higher mathematics they cover in the university courses. Of course, there are several Mathematical Practitioners in London who make a living tutoring people who want to learn the math - but you have neither the time nor the resources that solution would require.

As you work your way into the heart of London, it dawns on you that there may be a book that covers the topic - and does so in a way that you could follow. Caught up in this inspiration, you leave your coach as you pass St. Paul's Church - because you recall that the area around this churchyard is famous for the many bookshops to be found there.

Each shop has a sign hanging over its doorway. You see the sign of the Pope's Head, the sign of the Crossed Daggers, and finally - the sign of the Sun. Clearly, this must be the place.

You enter and find that the shop has a good selection of books on dialing - but they are all rather thick tomes. They would all be fairly difficult to work through - and frankly they seem to assume more math than you have at your command.

Disappointed, you are about to leave, when Thomas Pierrepont, the proprietor of the sign of the Sun, greets you and offers to help. You let him know what you want, and he points you to a small volume you had overlooked on the shelves.

You open the book to find a very ambitious title for such a small work: ***Dialling Universal***, by George Serle, *Practicioner in the Mathematicks*. Serle promises "*an easie and most speedy way*" to solve your problem by showing you how to use the scales on a small, portable ruler.

Thomas Pierrepont shows you an example of the rulers that are sold with the book - and he notes that they are made by Anthony Thompson, who in this year of 1657 is the preeminent maker of mathematical instruments in London.

You are sold! But just as you reach into your pocket to pay for the book and ruler, the Ghost of Dialing Past taps you on the shoulder and announces that you have to return to the hectic life you lead in the twentieth century.

Reluctantly, you put the book down and follow the Ghost. But as you return, you manage to take an occasional look into the time stream as the years tick by - and in this way, you can roughly follow the fortunes of this dialing scale concept.

1638 Samuel Foster publishes *The Art of Dialling*, providing the first printed account of Latitude and Hour Scales.

1658 George Serle and Anthony Thompson team up to produce a full set of scales on a ruler, together with an instruction booklet *Dialling Universal*.

1659 Foster's friend John Twysden publishes Foster's *Miscellanies*, a book which includes "*Demonstratio Quadrantis Horometrici*" (*Demonstration of an Horometrical Quadrant*) from Foster's notes, showing the mathematical development of the scales.

1664 Walter Hayes makes and sells the "Dialling Scales."

1669 Captain Samuel Sturmy publishes *The Mariners Magazine*, featuring a geometrical development of all the scales, instructions on their use, and the claim that Philip Staynred, a Bristol surveyor, invented them in 1632. (Staynred never publishes any description of the scales.)

1700 Thomas Tuttell, instrument maker to the King, makes and sells "Serle's Dialling Scales."

1760 James Ferguson (*Lectures on Select Subjects - X*) details the construction of the scales of latitudes and hours "which may be had on scales commonly sold by mathematical instrument makers," noting that "This is the easiest of all mechanical methods, and by much the best."

1812 William Johnson and Thomas Exley repeat Ferguson's discussion of scales in *The Imperial Encyclopedia*.

1852 Henry Meikle produces a comprehensive survey article on gnomonics for the *Encyclopaedia Britannica* (8th edition), noting that "we may lay down the hour lines by means of a dialling scale, the easiest method of any." The survey covers instructions for the use and design of scales.

1875 Hugh Godfray authors the dialing article for the *Encyclopaedia Britannica* (9th edition), with no discussion of dialing scales.

1969 Frank Cousins publishes *Sundials*, which includes a discussion of dialing scales: "At the time of writing I have not discovered an explanation of these scales in the extensive literature known to me."

Cousins "rediscovers" the theory behind the scales by examining a turn of the century dialing scale accompanied by a "crude typewritten booklet"; he notes that "No indication of the genesis of the ingenious scales is given and the reader is left to marvel at their versatility and the essential nature of their success."

1995 The North American Sundial Society sponsors publication of an annotated new edition of George Serle's ***Dialling Universal*** and the production of an accompanying Ruler! (See ***SPECIAL OFFER***)

This book and ruler combination provide a graphical method to lay out the hour lines of a sundial on any plane - horizontal, vertical, inclining, declining, etc. The metal ruler is 6 inches in length. The scales are also reproduced on paper in the book itself.

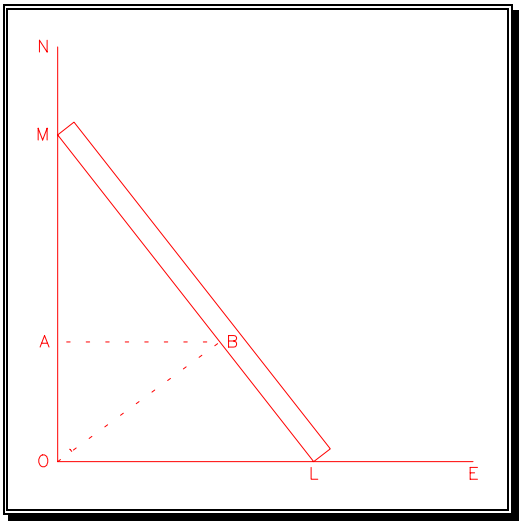
To see how the process works for a horizontal dial, suppose we begin with two perpendicular lines NO and OE, oriented to the north and to the east, respectively. Two of the scales on the ruler are marked Lat and Hour. Lay the Lat scale along the line OE, with the origin at O, and mark the point L on OE corresponding to the intended latitude of the dial.

Now lay the Hour scale so that its 6 o'clock end point lies on the point L and the 0 o'clock end point M lies on the line NO. Mark the hour points B on the dial face as determined by this Hour scale. Draw the required hour lines by connecting each of these points to the origin.

This completes the afternoon portion of the dial. Repeat the process on the west side for the morning hours, and extend hour lines through the origin or repeat in the other quadrants to finish the dial.

The procedure is very simple, and it can be used effectively to "reverse engineer" a dial as well - *i.e.* to determine the latitude for which a given dial was designed. See Steven Woodbury's article.

The secret of the dialing scales is that it is possible to turn the standard sundial math (the problem of constructing an angle Z such that tan Z = tan t sin φ) into the task of drawing a special right triangle which separates the latitude measure into the sides of the triangle and the hour measure into the hypotenuse. When the task no longer combines latitude and hours in one equation, we can design scales to measure out the right lengths - one for latitude ($OL = \sin\varphi \Big/ \sqrt{1+\sin^2\varphi}$) and one for time ($MB = \sin t \Big/ (\cos t + \sin t)$). The following proof demonstrates that this procedure indeed does work:



Given perpendicular lines *NO* and *OE*, aligned to the north and east respectively, lay the *Lat* scale along

OE and mark the point *L* corresponding to the dial's intended latitude; the length *OL* is $\sin\varphi / \sqrt{1+\sin^2\varphi}$. Place the *Hour* scale so that its entire length lies between *L* and a point *M* on the north line; the length *LM* is therefore 1, and by the Pythagorean theorem, the line *MO* is $1/\sqrt{1+\sin^2\varphi}$.

The point on the *Hour* scale *ML* corresponding to time *t* is *B*. By the design of the *Hour* scale, we know the length *MB* is $\sin t / (\cos t + \sin t)$; and the resulting complementary length *BL* is $\cos t / (\cos t + \sin t)$. The hourline for time *t* is *OB*.

To prove that this construction is valid, it suffices to establish that $\tan AOB = \tan t \sin\varphi$, thus satisfying the familiar formula for a horizontal dial.
Triangles *MOL* and *MAB* are similar, so $AB / MB = OL / ML$, and $AO / MO = BL / ML$. Given that *ML* = 1, we obtain the desired result as follows:

$$AB = MB \times OL = \sin t \sin\varphi \Big/ (\cos t + \sin t)\sqrt{1+\sin^2\varphi}$$

$$AO = MO \times BL = \cos t \Big/ (\cos t + \sin t)\sqrt{1+\sin^2\varphi}$$

and $\tan AOB = AB / AO = \tan t \sin\varphi$.

What Latitude Was It Designed For?

Steven Woodbury

In your rambles as a dial hunter you come across an interesting sundial installed in a garden. Your protractor or inclinometer shows that the gnomon makes the correct angle for your latitude. But the gnomon looks like a replacement, and you want to know whether the dial plate was designed for that latitude. You didn't bring your calculator ... or maybe you can never quite remember the trigonometry.

Serle's dialing scales can solve your problem. They are designed for constructing new dials, but with a little "reverse engineering" they can be used to determine the latitude for which an existing dial plate was originally designed. (These instructions are for the common case of a horizontal or south-facing vertical dial, with a flat dial plate, and no longitude correction.)

Step 1. On a tracing or rubbing of the dial plate, draw the "six o'clock" line, connecting 6:00 AM and 6:00 PM. Draw the "twelve o'clock" line (at a right angle). (Usually this is the edge of the base of the gnomon.) From the intersection of these two lines to the hour marks draw radial hour lines. Note that if the design of the dial plate already incorporates these lines, the tracing will not be necessary, and the reading can be taken directly from the dial.

Step 2. Place Serle's "Hour" scale on the tracing, with the zero end on the "twelve o'clock" line, and the "6" end on the "six o'clock" line. Slide and pivot the scale, keeping the end points on these lines, until the hour marks on the scale make the best match with the radial hour lines on the dial plate (see Figure 1). Mark the point on the "six o'clock" line where the hour scale touches it.

Step 3. Use Serle's "Lat." scale to measure along the "six o'clock" line from the "twelve o'clock" line to your mark. This gives a direct reading of the latitude for which the horizontal dial was designed (see Figure 2). If you are measuring a direct south vertical dial, this result is the complement of the latitude (*i.e.* latitude = 90 degrees minus this result).

I have made copies of Serle's dialing scales in several different sizes on acetate and heavy paper, which I carry in my "dial hunter's kit" for this purpose.

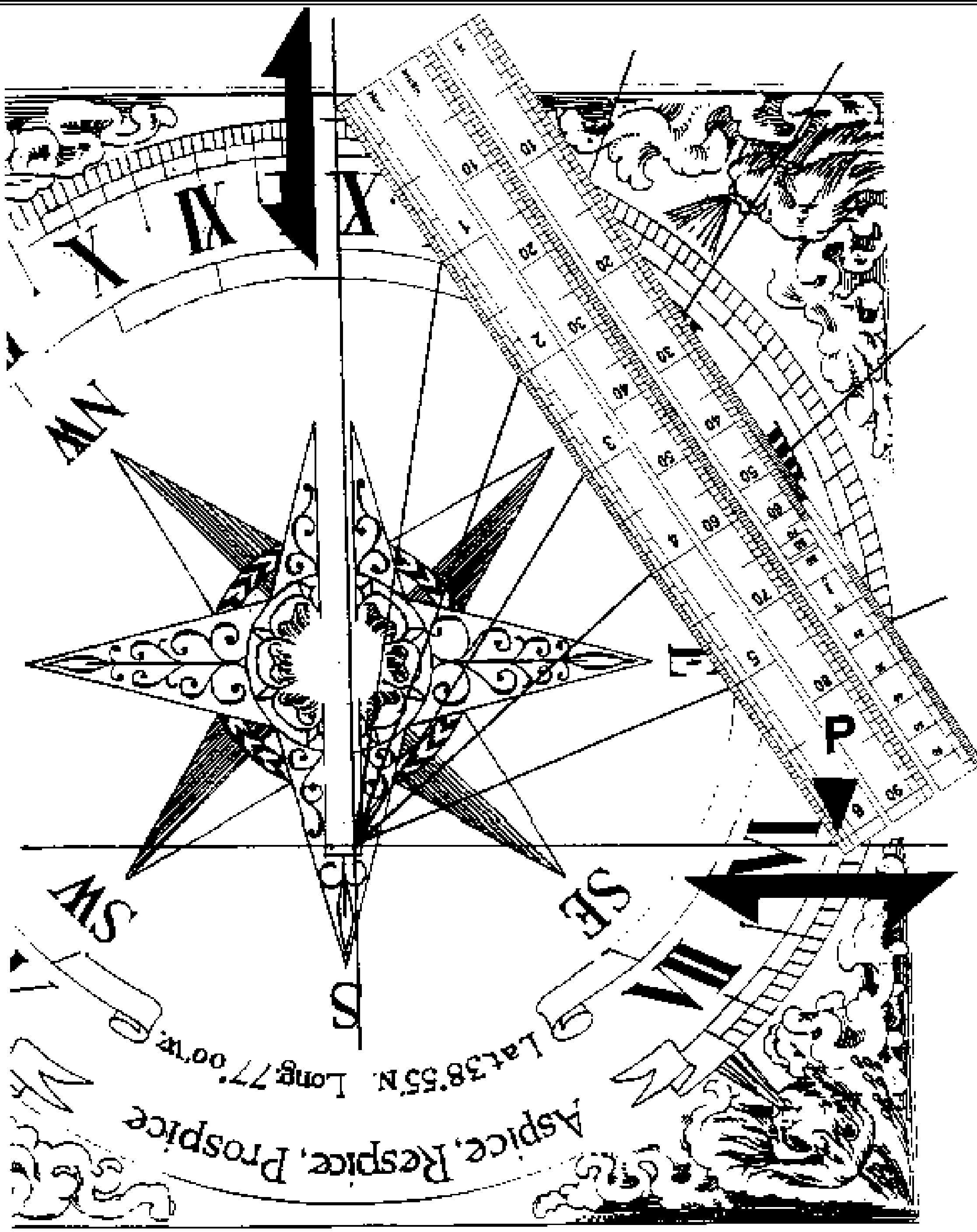


Figure 1

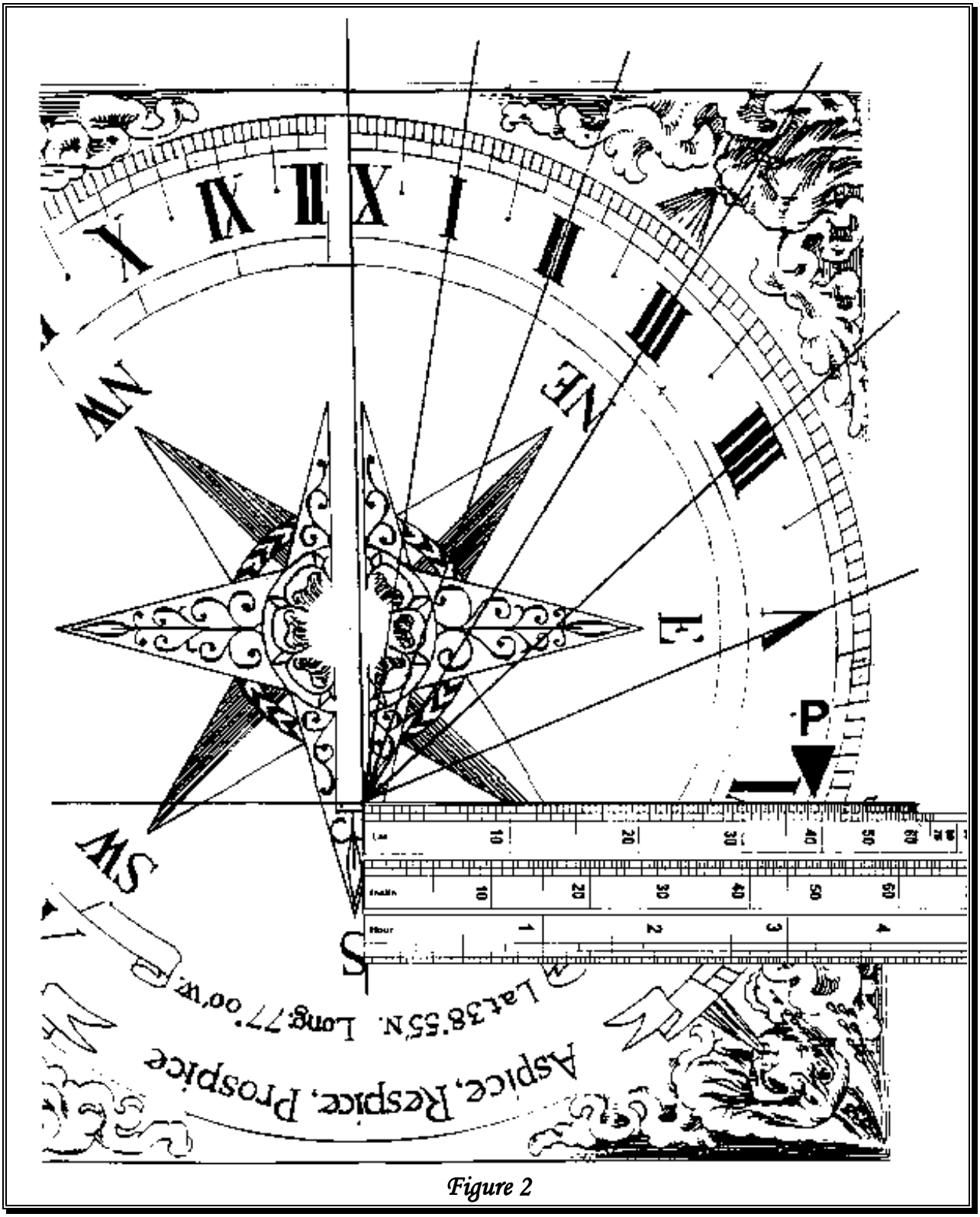

XII I II III IV V VI VII VIII IX X XI
NW
NE
E
P
S
SW
Aspice, Respice, Prospice
Lat 38.55N. Long 77.00W

Figure 2